Abstract

In this note we describe the fitting algorithm and the selection criteria strategies we have developed, in order to improve the tracking resolution of the NESTOR detector deployed in March of 2003. Monte Carlo events are used to demonstrate that the zenith angle estimation of the reconstructed tracks has a mean value of 8.5 degrees, while the three-dimensional angular deviation exhibit a mean value of 14 degrees.

1. Introduction

The NESTOR detector deployed in March of 2003 [1,2] was a fully equipped NESTOR floor along with many environmental systems attached to the bottom station (Figure 1).

The detector consisted of 12 photomultiplier tubes (PMTs) [3] and the corresponding electronics, which were responsible for triggering, digitization and data transmission to shore. The electronics were housed inside a large titanium sphere (1m
in diameter) placed at the center of the hexagonal floor, while the PMTs were arranged in pairs (one pair per arm) with one PMT looking up and the other down. The detector operation commands and the digitized data were transmitted via a 30 km long electro-optical cable, which was also used for power supply of the detector.

During operation, the Data Acquisition system [4] of the NESTOR detector stores the accumulated data packets into data files containing the digitised pulses and the operational and environmental parameters of the detector. The signal processing algorithms decode these data files and restore the original form of the pulses taking into account the response function of the system and the calibration data. Finally, the raw data processing results in the accurate determination of the arrival time and the height of the pulses [5], which are used by the tracking algorithm in order to estimate the 5 track parameters (zenith angle, azimuth angle and the pseudo-vertex position).

2. Fitting Algorithm

In the first stage of the track reconstruction, the fitting algorithm makes use of all the PMT pulses lying inside the coincidence time window (hits). The coincidence time window is chosen to be the time needed by a photon to cover the maximum distance, $d_{\text{max}}$, between two optical sensors ($t_{\text{max}} = \frac{d_{\text{max}}}{c}$, where $n_{\text{min}}$ is the minimum value of the refractive index in the water within the wavelength spectrum of the Cherenkov emission). For the detector geometry used in the 2003 Run the coincidence time window was defined as 60 ns. However, when a PMT waveform consists of multiple hits, all within the coincidence time window, only the higher amplitude pulse is considered. The other hits on this PMT are used in cases where the reconstruction procedure does not converge or when the selected hit is rejected during the second reconstruction stage (see below).

The arrival times of the selected hits are used in a $\chi^2$ minimization in order to estimate the track parameters and evaluate the error matrix of the estimates. The $\chi^2$ estimator is defined (see also Figure 2) as:

$$\chi^2 = \sum_{i=1}^{N_{\text{hit}}} \left( \frac{t_{\text{exp}} - t_{\text{data}}}{\sigma_{\text{data}}} \right)^2$$
Figure 2

Geometrical representation of the transmission of Cherenkov light to a PMT.

**Track Parameters**

θ: zenith angle
φ: azimuth angle
(Vx,Vy,Vz): pseudo-vertex coordinates
(Vx’,Vy’,Vz’): pseudo-vertex

where:

N_{hit} is the number of the hits used for the track reconstruction

\( t_i^\text{exp} \equiv t_i^\text{exp}(\theta, \varphi, V_x, V_y, V_z) \) is the expected arrival time of the \( i^{th} \) hit, assuming that the pulse is the PMT response to the Cherenkov light produced by a muon track with zenith angle \( \theta \), azimuthal angle \( \varphi \) and pseudovertex\(^1\) coordinates \{V_x, V_y, V_z\},

\( t_i^\text{data} \) is the measured arrival time of the \( i^{th} \) hit,

\( \sigma_i^\text{data} \) is the resolution in measuring the arrival time of the \( i^{th} \) hit. This has been measured for each PMT as a function of the pulse amplitude, both in the laboratory and in the deep sea [6].

The \( t^\text{exp} \) is evaluated by means of the geometrical elements shown in Figure 2, as:

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\(^1\) This is a point on the muon track corresponding to the start of the experimental time window (0-465ns).
\[
I_{\text{exp}} = \frac{d_m}{c} + \frac{d_{\gamma}}{(c/n)} = \frac{(L + d \times \tan \theta_\gamma)}{c},
\]

\[
L = \cos \phi \times \sin \theta \times (x - V_x) + \sin \phi \times \sin \theta \times (y - V_y) + \cos \theta \times (z - V_z),
\]

\[
d = \sqrt{[(x - V_x) - L \times \cos \theta]^2 + [(y - V_y) - L \times \sin \phi]^2 + [(z - V_z) - L \times \cos \theta]^2}
\]

where \(\{x, y, z\}\) are the coordinates of the PMT centre.

The minimization procedure can often converge to multiple solutions\(^2\), each corresponding to a candidate track. Figure 3 presents the \(\chi^2\) probability distribution of the candidate tracks from a sample of Monte Carlo events. As it is shown there is an excess of events at lower probabilities due to false track candidates.

![Figure 3](image)

*Figure 3*

*The \(\chi^2\) probability distribution of Monte Carlo samples, at the end of the first reconstruction stage.*

The second reconstruction stage is an iterative attempt to improve the tracking resolution for each candidate found. The algorithm rejects the hit with the largest contribution to the \(\chi^2\) value and tests if this improves the \(\chi^2\) probability. Also in this stage, cases where multiple hits on a PMT have been recorded within the coincidence time window, the hits with lower pulse height can be considered.

\(^2\) tracks with almost the same \(\chi^2\) probability.
3. Candidate Selection

Only those candidate tracks with a $\chi^2$ probability greater than 0.1 are retained for further analysis. For each track candidate, the expected quantity of Cherenkov light reaching each of the PMT’s in the detector can be compared to the measured pulse heights observed. Criteria can thus be established to further refine the trajectory selection.

A significant fraction of the events give more than one possible track solution from the pulse arrival time analysis alone. This is due to an inherent geometrical degeneracy (mirror solution), because there is symmetry between the Cherenkov light cones emitted from tracks that form an angle of twice the Cherenkov angle (Figure 4).

![Figure 4](image)

*Figure 4*

*A pictorial representation of the light wave front coming from two tracks with an angle between them equal to the twice of the Cherenkov angle. The wave fronts coincide in the region of the detector.*

This ambiguity can be resolved by examining the light intensity distribution on the PMTs.

To quantify this light distribution, a photon-likelihood, $L_{ph}$, is defined as:
\[ L_{ph} = \prod_{i=1}^{N_{hit}} P_i(V_{\text{data}}; \mu_{\text{exp}}) \]

where, \( P_i(V_{\text{data}}; \mu_{\text{exp}}) \) is the probability that the pulse height of the \( i^{th} \) hit is \( V_{\text{data}} \) when the expected mean number of photoelectrons emitted from the photocathode due to Cherenkov light, is \( \mu_{\text{exp}} \).

The expected mean number of photoelectrons, \( \mu_{\text{exp}} \), is calculated, taking into account the candidate track parameters and the PMT positions, the mean number of Cherenkov photons produced per unit track length, the PMT geometric cross section, the light absorption from the water and other materials, the PMT collection and quantum efficiency. The probability \( P_i(V_{\text{data}}; \mu_{\text{exp}}) \) is then estimated as the convolution of the poissonian probability function, of mean value \( \mu_{\text{exp}} \), for the emission of \( n \) photoelectrons from the photocathode with the probability density function \( R_i(V_{\text{data}}; n) \). This function, \( R_i(V_{\text{data}}; n) \), expresses the probability of the \( i^{th} \) hit to have a pulse height equal to \( V_{\text{data}} \), assuming that the pulse is produced from the emission of \( n \) photoelectrons. It is the pulse height distribution (normalized to unity), which corresponds to \( n \) photoelectrons and it has been evaluated for each of the PMTs by the convolution of \( n \) one-photoelectron pulse height distributions\(^3\).

In Figure 5 is shown the function \( R_i(V_{\text{data}}; n) \) for \( n=1, 2 \) and 3.

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\(^3\) The one-photoelectron pulse height distribution of each PMT has been measured in the laboratory, before the detector deployment, and it has been verified with the calibration data collected in the sea.
Consequently the probability function $P_i(V_{\text{data}}; \mu_{\exp})$ is expressed as:

$$P_i(V_{\text{data}}; \mu_{\exp}) = \sum_{n=1}^{\infty} \frac{(\mu_{\exp})^n e^{-\mu_{\exp}}}{n!} R_i(V_{\text{data}}; n)$$

Figure 6 plots the distribution of the values of the negative logarithm of the photon-likelihood ($-\ln L_{ph}$) with respect to the difference between true and estimated zenith angle for Monte Carlo candidate tracks that survive the second reconstruction stage. In order to improve the reconstruction resolution, candidate tracks with negative logarithms of the photon-likelihood greater than 16 are rejected. For the remaining multiple track candidates in an event, a track is selected if the $\chi^2$-probability exceeds the $\chi^2$ probability of the other candidate(s) by more than 0.1 or else has a lower $-\ln L_{ph}$ value.
The negative logarithm of the photon-likelihood with respect to the difference between true and estimated zenith angle for Monte Carlo candidate tracks that survive the second reconstruction stage.

An example demonstrating the discrimination power of the photon Likelihood is shown in Figure 7. The figure represents the digitized waveforms of the 12 PMTs for a Monte Carlo event generated according to the Okada model [7], which describes the atmospheric muon flux at the detector depth. There are six pairs of charts each pair corresponding to an arm of the detector. The upper chart of a pair represents the up-looking PMT and the lower the down-looking PMT. It is also shown the coincidence time window (dotted vertical lines) and the arrival time of each pulse (solid vertical lines). The heights of the solid vertical lines correspond to the height of the pulses. By selecting the pulses that lie inside the coincidence time window, the $\chi^2$ fitting procedure gives two candidate solutions. The $\chi^2$ value in both cases is equal to 0.171 for one degree of freedom. By inspecting the negative logarithm of the Photon Likelihood we conclude that the most probable event is the second one with a zenith angle 22°±7 degrees consistent with the value of the zenith angle (20.3 degrees) which this simulated muon has been generated.
Figure 7

The produced waveforms of a Monte Carlo event generated according to the Okada Model. The $\chi^2$ fit taking account the arrival times of the pulses, ends up with two candidate solutions. The estimated track parameters (zenith ($\theta$), azimuth ($\phi$) angle and pseudovertex position ($V_x, V_y, V_z$)) along with the impact parameter ($d$), the Correlation Matrix and the $\chi^2$ values are displayed for both cases. The most probable track is the one with the lower negative value of the logarithm of the Photon Likelihood ($QL$).
4. Global Selection Criteria

In order to improve the reconstruction resolution, global selection criteria must be applied to the data sample surviving the second stage of the reconstruction algorithm. These criteria concern:

- The impact parameter of the reconstructed track.
- The charge collected from the photomultiplier tubes.

By requiring the impact parameter$^4$ of the reconstructed track to exceed the detector radius of 6m (Figure 8a), cases that cannot be resolved correctly because of the small lever arm of the test detector are excluded. In addition, by selecting tracks with more than $4 \cdot N_{hit}$ photoelectrons$^5$ (Figure 8b) the discrimination power of the photon-likelihood criterion is increased.

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Figure 8

*Distributions of the impact parameter (above) and the total number of photoelectrons per track (below). The arrows indicate the applied cuts (threshold values) in order to increase the estimation accuracy. The number in each arrow in (b) indicate the level of coincidence.*

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$^4$ This is the perpendicular distance of the reconstructed track from the center of the Ti-floor

$^5$ This is the sum of the pulse height of the hits (in units of the mean value of the one-photoelectron pulse height distribution) used in the fit.
5. Tracking Performance

Using the Monte Carlo event sample, the ways in which the various selection criteria affect the resolution of the zenith angle estimation have been studied. It can be seen in Figure 9 that these selection criteria, applied throughout the reconstruction stages, reject the majority of badly fitted track candidates, especially the ghost “mirror” tracks.

The distribution of the difference between the reconstructed and the “true” zenith angle for the selected Monte Carlo tracks (Figure 9a) exhibits a central Gaussian peak of a sigma equal to 8.5°.

![Figure 9](image.png)

*Figure 9*

Demonstration of the effect of the selection criteria on the resolution in reconstructing the zenith angle from the Monte Carlo event sample. a) The resolution distribution of the selected track sample. b) The resolution distribution without the photon-likelihood selection criteria. c) The resolution distribution without the impact parameter selection criteria. d) The resolution distribution without the selection criteria on the total number of photoelectrons per track.
In order to check that the track reconstruction algorithm gives consistent error estimations, the pull distribution for the reconstructed zenith angle was generated using the Monte Carlo event sample. The pull distribution is the deviation of the reconstructed zenith angle from its true value divided by the estimated error. Figure 10 shows that the pull distribution exhibits Gaussian shape with a mean value and sigma consistent with zero and one respectively.

![Figure 10](image)

*Figure 10*

The pull distribution of the reconstructed zenith angles of Monte Carlo produced tracks.

Finally, Figure 11 presents the three-dimensional angular deviation of the reconstructed tracks from their true direction, quantified using the Monte Carlo event sample. After applying the track selection criteria described above, more than 90% of the reconstructed tracks exhibit a mean angular deviation of 14°, whilst the remaining events are concentrated at high values from mirror solutions.
6. Conclusions

In this note we described the reconstruction algorithms we have used in order to estimate the atmospheric muon track parameters with the NESTOR test detector deployed in March 2003. It was demonstrated that the resolution in the zenith angle determination, which is achieved with the standard fitting algorithms (based on the timing of the PMT pulses), could be significantly increased with the inclusion of the collected light intensity into the fitting procedure. This way the majority of the mirror solutions could be resolved. Furthermore two global selection criteria based on the estimated impact parameter and the charge collected by the PMTs were used in order to increase the resolution and the reconstruction efficiency. As a result the zenith angle estimation of the reconstructed tracks was estimated to have a mean value of 8.5
degrees, while the three-dimensional angular deviation was estimated to have a mean value of 14 degrees.

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References


